Vector Coupled-Mode Calculation of Guided Vector Modes on an Equilateral Three-Core Optical Fiber

Horng Shou Huang and Hung-Chun Chang, *Member, IEEE*

Abstract—**The guided vector modes propagating on an equilateral three-core optical fiber, in which the component cores are identical, single-moded, and arrayed in an equilateral triangle, are determined using the coupled-mode approach. It is shown that in the case of weakly guiding fibers, the polarization patterns of the six vector array modes can be correctly obtained if the vectorial-form coupled-mode theory is applied and the coupling among the six HE_{11} modes of the individual cores is considered.**

I. Introduction

THREE-CORE optical fibers, or three-core fiber couplers, having a configuration in which the component cores are identical, single-moded, and arrayed in an equilateral triangle have been used as 3×3 directional coupling and power dividing devices in fiber interferometers [1]–[3] and recently in optical homodyne receivers [4]. In the theoretical aspect, the modal properties and scattering matrix of such equilateral three-core fibers have been studied by various authors [5]–[7]. The guided vector modes on this kind of three-core structure were discussed using symmetry arguments [5] and a perturbation method [6]. However, their results for the polarization patterns and the mode splitting property were distinctly different. We have recently performed a rigorous analysis of the propagation characteristics of the equilateral three-core fiber using the circular harmonics expansion method and the results confirm the prediction of the perturbation method [8].

In this letter we show that the six vector modes guided on a weakly-guiding equilateral three-core fiber can be determined correctly based on a simple coupled-mode calculation. The coupled-mode theory in the vectorial form is employed and the coupling among six HE_{11} modes (two for each core), that are the exact vector modes on individual cores, is thus considered. We have derived closed-form formulas for the coupling coefficients between two HE_{11} modes [9], so that the coupled-mode calculation involving the complex vector modes can be easily carried out and the numerical calculations can be highly accurate within the coupled-mode formalism. In Section II we describe the coupled-mode formulation. Numerical examples and some related discussions are given in Section III. The conclusions are drawn in Section IV.

Manuscript received November 5, 1990. This work was supported by the National Science Council of the Republic of China under Grants NSC78-0417-E002-01 and NSC79-0417-E002-01.

The authors are with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan 10764, R.O.C.

IEEE Log Number 9042600.

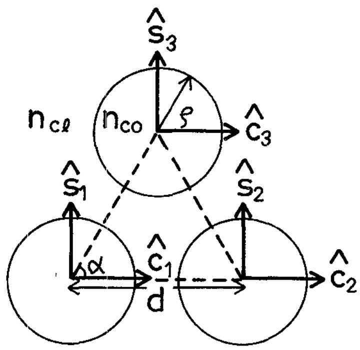

Fig. 1. Cross section and coordinate systems of an equilateral three-core fiber. Angle α is 60°.

II. The Coupled-Mode Formulation

Consider the equilateral three-core structure with its cross section depicted in Fig. 1. The three identical round optical fiber cores are embedded in a uniform cladding medium. The refractive indices of the core and cladding are denoted by n_{co} and n_{cl}, respectively, the radius of the core by ρ, and the center-to-center core separation by d. The angle α in Fig. 1 is 60°. Three different coordinate systems are defined as shown in Fig. 1 and the unit vectors along the axes of each system are denoted by $\hat{s}_i$ and $\hat{c}_i$, for $i = 1$, 2, and 3. On the ith core, when in isolation from the others, there are two orthogonally polarized HE_{11} modes: the E_z-sine mode for which the total electric field $\bar{E}_i^s$ is predominantly polarized in the s_i direction, and the E_z-cosine mode for which the electric field is predominantly polarized in the c_i direction. Therefore, there are totally six guided modes involving in the coupling. They are coupled together to form six array modes on the whole structure.

Before writing down the coupled-mode equations for the equilateral three-core fiber, we first determine the coupling coefficients between the cores. Let K_c be the coupling coefficient between the modes $\bar{E}_1^c$ and $\bar{E}_3^c$, K_s the coupling coefficient between the modes $\bar{E}_1^s$ and $\bar{E}_3^s$, and K_{cs} the coupling coefficient between the modes $\bar{E}_1^c$ and $\bar{E}_3^s$. These coefficients are defined as in [10]. Since the E_z-sine and E_z-cosine modes for each core are degenerate, we are able to construct two modes with electric fields

$$\bar{E}_i^x = \bar{E}_i^c \cos 60^\circ + \bar{E}_i^s \sin 60^\circ \tag{1a}$$

and

$$\bar{E}_i^y = -\bar{E}_i^c \sin 60^\circ + \bar{E}_i^s \cos 60^\circ, \tag{1b}$$

Respectively, where $i = 1$ or 3. It can be shown that $K_c = C_x \cos^2 60° + C_y \sin^2 60°$, $K_s = C_x \sin^2 60° + C_y \cos^2 60°$, and $K_{cs} = (C_x - C_y) \cos 60° \sin 60°$, where

$$C_x = \frac{\omega \epsilon_0}{2} \int \int_{A_c} (n_{co}^2 - n_{cl}^2) \bar{E}_1^x \cdot \bar{E}_3^{x*} \, ds \qquad (2a)$$

and

$$C_y = \frac{\omega \epsilon_0}{2} \int \int_{A_c} (n_{co}^2 - n_{cl}^2) \bar{E}_1^y \cdot \bar{E}_3^{y*} \, ds. \qquad (2b)$$

In (2a) and (2b), ω is the wave frequency, ϵ_0 is the permittivity of free space, A_c denotes the cross-sectional area of either core 1 or core 3, the asterisk represents complex conjugation, and all the electric fields are normalized. C_x and C_y can be evaluated using the analytical formulas given in [9].

According to the coupled-mode theory of [10], the coupled-mode equations for the system in Fig. 1 are of the form

$$\frac{d}{dz} \bar{a}(z) = j\overline{\overline{M}}\bar{a}(z) \qquad (3)$$

with $\exp(-j\omega t)$ field variation assumed. In (3), $\bar{a} = (a_1^c \; a_2^c \; a_3^c \; a_1^s \; a_2^s \; a_3^s)^T$, where a_1^c denotes the modal amplitude of the E_z-cosine mode on the first core, etc., and T represents transposition, and

$$\overline{\overline{M}} = \begin{bmatrix} \beta & C_x & C_1 & 0 & 0 & C_3 \\ C_x & \beta & C_1 & 0 & 0 & -C_3 \\ C_1 & C_1 & \beta & C_3 & -C_3 & 0 \\ 0 & 0 & C_3 & \beta & C_y & C_2 \\ 0 & 0 & -C_3 & C_y & \beta & C_2 \\ C_3 & -C_3 & 0 & C_2 & C_2 & \beta \end{bmatrix} \qquad (4)$$

where β is the propagation constant of the HE_{11} mode, $C_1 = \frac{1}{4}C_x + \frac{3}{4}C_y$, $C_2 = \frac{3}{4}C_x + \frac{1}{4}C_y$, and $C_3 = \sqrt{3}/4 \, (C_x - C_y)$. The eigenvalues of $\overline{\overline{M}}$ are the propagation constants of the array modes on the coupled-core structure and the corresponding eigenvectors tell how the individual modes are combined to form the array modes. More specifically, by summing up the six individual vector modes according to the weighting specified by the components of the eigenvector, we obtain an array mode, which is of course vector mode.

Due to the symmetry of the coupled structure, there are three even modes and three odd modes, with the z (longitudinal) component of the electric field being symmetric and antisymmetric with respect to the s_3-z plane, respectively. The six array modes are designated as ψ_0^e (the fundamental even mode), ψ_1^e (the first of the first-order even modes), ψ_2^e (the second of the first-order even modes), ψ_0^o (the fundamental odd mode), ψ_1^o (the first of the first-order odd modes), and ψ_2^o (the second of the first-order odd modes). The corresponding propagation constants, i.e., the eigenvalues of $\overline{\overline{M}}$, are found to be $\gamma_0^e = \gamma_0^o = \beta + \frac{1}{4}(C_x + C_y + \xi)$, $\gamma_1^e = \beta - \frac{3}{2}C_x + \frac{1}{2}C_y$, $\gamma_1^o = \beta + \frac{1}{2}C_x - \frac{3}{2}C_y$, and $\gamma_2^e = \gamma_2^o = \beta + \frac{1}{4}(C_x + C_y - \xi)$, where $\xi = (13C_x^2 + 10C_xC_y + 13C_y^2)^{1/2}$. Thus, ψ_0^e and ψ_0^o are degenerate and so are ψ_2^e and ψ_2^o.

III. Numerical Examples and Discussion

Detailed analysis shows that for equilateral three-core fiber couplers, the normalized propagation constant $\rho\gamma$ of each array mode is uniquely determined once the normalized frequency $V = (2\pi/\lambda)\rho(n_{co}^2 - n_{cl}^2)^{1/2}$ (λ being the light wavelength), the profile height parameter $\Delta = (1 - n_{cl}^2/n_{co}^2)/2$, and the normalized separation $D = d/\rho$ are given. In the following discussion, we take $D = 2$, i.e., the adjacent cores in Fig. 1 are touched. For $\Delta = 0.005$ and 0.05, we have calculated $\rho\gamma$ as a function of V for each of the six modes by solving for the eigenvalues of $\overline{\overline{M}}$ and compared the result with that obtained using the exact method [8], and found that the maximum errors in $\rho\gamma$ in the coupled mode calculation are 0.015% ($\Delta = 0.005$) and 0.17% ($\Delta = 0.05$), respectively. Therefore, for weakly guiding fibers (Δ small), the formulation described in the previous section is highly accurate in determining the array mode propagation constants.

Our main concern is to see if the coupled-mode formalism can give the correct polarization patterns for the array modes. First, consider a weakly guiding case with $n_{co} = 1.458$, $n_{cl} = 1.455$, $\rho = 4.5$ μm, and $\lambda = 1.3$ μm, and thus $\Delta = 0.002$. The patterns of the six array modes as constructed from eigenvectors are shown in Fig. 2. For each mode, we use six arrows to represent the orientation and relative strength of the electric fields at the three core centers and the three tangent points on the core/cladding boundary. For this case, we have found that the patterns are identical with (more precisely speaking, indistinguishable from) those given in [9] based on the more exact circular harmonics expansion method. This indicates that for the weakly guiding equilateral three-core fiber coupler, the coupled-mode analysis considering the polarization effect (i.e., using the exact HE_{11} modes) can provide correct description of the vector array mode polarization patterns.

As Δ increases, the fibers become more strongly guiding and the coupled-mode analysis would be expected to decrease its own accuracy. To check the dependence of this accuracy on Δ, we have examined two cases with $\Delta = 0.05$ and $\Delta = 0.25$ ($n_{cl} = 1.455$ in both cases), respectively, and keeping the other parameters the same. For $\Delta = 0.05$, deviations of the coupled-mode predictions from the exact solutions for the electric fields at the six positions are found to be at most 20% in strength and 3° in orientation. For $\Delta = 0.25$, the polarization patterns obtained by the coupled-mode method are found to become highly distorted as compared with exact patterns, although the errors in the coupled-mode results of the propagation constants of the array modes for this case are less then 3%.

We have also formulated the vector coupled-mode calculation for the equilateral three-core fiber using the recently developed new coupled-mode theory [11]. Its predictions of the polarization patterns for the vector modes in the weakly guiding case, e.g., the case of Fig. 2, are again correct and indistinguishable from the exact results. By concerning the complexity of the methods, the simpler conventional coupled-mode theory as employed in this letter is apparently preferred for determining the polarization patterns on a weakly guiding structure. Details of the study of the equilateral three core system based on the new coupled-mode theory will be reported separately.

IV. Conclusion

Based on a simple coupled-mode calculation, we have successfully obtained the correct polarization patterns of the vector array modes propagating on a weakly guiding equilateral three-core optical fiber. The conventional coupled-mode theory in the vectorial form is employed and the coupling among six exact HE_{11} modes (two for each core) is considered in the calculation. It is found that the coupled-mode analysis decreases its own

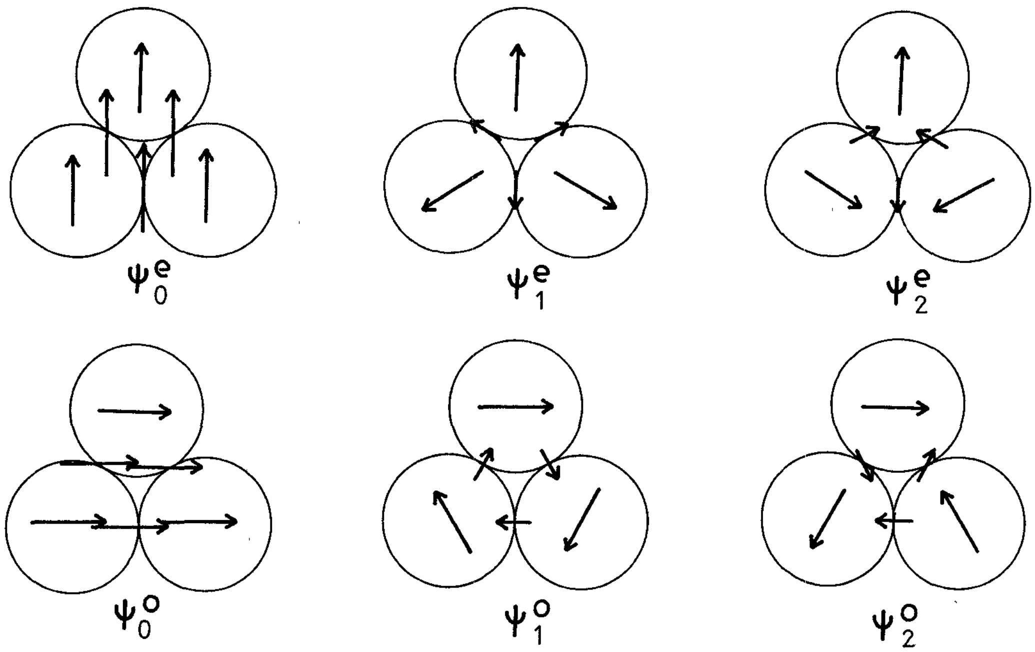

Fig. 2. Coupled-mode predictions of polarization patterns for six vector modes of weakly guiding ($\Delta = 0.002$) equilateral three-core fiber. Patterns are indistinguishable from those obtained using exact method (not shown).

accuracy as the fibers become more strongly guiding and the predicted field patterns are distorted.

References

[1] S. K. Sheem, "Optical fiber interferometers with [3 × 3] directional couplers: Analysis," *J. Appl. Phys.*, vol. 52, pp. 3865–3872, 1981.

[2] S. K. Sheem, T. G. Giallorenzi, and K. Koo, "Optical techniques to solve the signal fading problem in fiber interferometers," *Appl. Opt.*, vol. 21, pp. 689–693, 1982.

[3] K. P. Koo, A. B. Tveten, and A. Dandridge, "Passive stabilization scheme for fiber interferometers using (3 × 3) fiber directional couplers," *Appl. Phys. Lett.*, vol. 41, pp. 616–618, 1982.

[4] T. D. Stephens and G. Nicholson, "Optical homodyne receiver with a six-port fiber coupler," *Electron. Lett.*, vol. 23, pp. 1106–1108, 1987.

[5] R. J. Black, L. Gagnon, R. C. Youngquist, and R. H. Wentworth, "Modes of evanescent 3 × 3 couplers and three-core fibers," *Electron. Lett.*, vol. 22, pp. 1311–1313, 1986.

[6] A. J. Stevenson and J. D. Love, "Vector modes of six-port couplers," *Electron. Lett.*, vol. 23, pp. 1011–1013, 1987.

[7] J. Pietzsch, "Scattering matrix analysis of 3 × 3 fiber couplers," *IEEE J. Lightwave Technol.*, vol. 7, pp. 303–307, 1989.

[8] H. S. Huang and H.-C. Chang, "Analysis of equilateral three-core fibers by circular harmonics expansion method," *IEEE J. Lightwave Technol.*, vol. 8, pp. 945–952, 1990.

[9] ——, "Analysis of optical fiber directional coupling based on the HE_{11} modes—Part I: The identical-core case," *IEEE J. Lightwave Technol.*, vol. 8, pp. 823–831, 1990.

[10] A. W. Snyder, "Coupled-mode theory for optical fibers," *J. Opt. Soc. Amer.*, vol. 62, pp. 1267–1277, 1972.

[11] H.-C. Chang, "Coupled-mode equations for dielectric waveguides based on projection and partition modal amplitudes," *IEEE J. Quantum Electron.*, vol. QE-23, pp. 1929–1937, 1987; references therein.